Thermal rub effect in rotating machines

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This article discusses the thermal/mechanical effect of rotor-to-stator rubbing in heavy rotating machines. A simple model explains the often-observed-in-the-field "spirals" in the 1X response at a constant rotative speed.

Introduction

Rotor-to-stator rubbing in rotating machines is a malfunction associated with a physical contact between rotating and stationary parts, which, during normal operation, should not be touching. This contact causes changes in the system force balance and dynamic stiffness and results in modifications of the machine motion. The effect is usually associated with a decrease in the energy provided to maintain the main operational motion (rotational mode), and an increase in the level of "parasitic" vibrations.

A rotor-to-stator rub is a secondary malfunction resulting from a primary cause which perturbs the machine during normal operational conditions. This primary cause can originate from various sources, such as unbalance, thermal and/or assembly misalignments, stator/casing motion, fluid-induced rotor self-excited vibrations due to fluid forces in bearings, and/or in the main flow, and in seals, in particular. The occurrence of the primary source is manifested by changes in rotor centerline position and/or shaft lateral vibrations so that allowable clearances are exceeded, and the rotor-to-stator contact occurs.

A rotor-to-stator rub involves several physical phenomena, the most important being friction, impacting, coupling and stiffening effects.

The rub-related rotor-to-stator friction is responsible for the rotor thermal balance change, known as "Newkirk effect." Based on Taylor's results [1], Newkirk [2] pointed out that, when a rubbing rotor is running below its first balance resonance speed, the rub-induced rotor lateral vibrations tend to increase in time. Later, this effect was studied by many other researchers, who confirmed that these vibrations can grow in amplitude and phase, resulting in "spiral" vibrations.

Thermal effect of friction

During machine operation, the rub of an unbalanced shaft usually takes place at the seal with the smallest clearance which is closest to the shaft antinodal position. Rotors can also rub against packings or oil deflectors. The rub occurs at the shaft "high spot" (angular location under the highest tension stress). At a constant rotative speed, for the mode of vibration predominantly synchronous with a circular or slightly elliptical Orbit, the rubbing high spot occurs at the same shaft location at each turn. The rub at the high spot causes friction-related heating and local thermal expansion of the shaft. Due to normal or accidental presence of fluids in the rotor/stator clearance areas, the rub-generated heat can be carried away by the fluid flow, so that the shaft heating and thermal expansion process may be relatively slow. Due to local expansion, the shaft bows, causing an additional unbalance in the rotor.

At a rotative speed lower than the first balance resonance, the phase angle between the high spot and the heavy spot (angular location of the unbalance) is less than 90 degrees (Figure 1); the additional bow-related unbalance then adds to the originally existing one (vectorially), and causes an increase of the overall unbalance inertia force acting on the rotor. The relationship between the new unbalance and the original one explains the phase lag growth, as the phase between the force and response adjusts to the value determined by the system parameters and a constant rotative speed. The phase lag increase causes the high spot to change, thus the rubbing location changes around the rotor circumference. The increase of total value of unbalance causes higher vibration amplitudes, which may result in stronger rubbing and larger amounts of heat. If the rub-related heating is fast enough, the rotor local thermal expansion becomes high and may exceed the elastic limit. The rotor then becomes

permanently bowed due to local plastic deformation and can no longer be operated.

More often, however, the process of rub-related, local thermal expansion and shaft bowing is slow, allowing for continuous phase adjustments. The resulting synchronous vibration amplitude changes are also slow. The phase lag growth may exhibit several full circles. This increasing phase lag reflects the continuous change of the high spot rubbing location. According to field reports, the periods of full circle phase lag growth may vary from a few minutes to over 24 hours (Figure 2).

Simple mathematical model of the thermal rubbing phenomenon

The effect of the rotor-to-stator thermal rub that is observed most often in rotating machines is the slowly increasing (or fluctuating) 1X vibration amplitude, with a definite growth of the 1X phase lag (Figure 2). The effect of this 1X amplitude variation and the phase growth can be explained using a simple mathematical model.

Consider the isotropic (symmetric) rotor of a fluid-handling machine at its first lateral mode. The rotor is rubbing against the stationary part at an axial location "R." The rubbing-related, local thermal expansion causes the rotor to bow. This results in an additional unbalance. The rotor model is as follows: (1)

$$M\ddot{z} + D_S\dot{z} + Kz + D(\dot{z} - j\lambda\Omega z) = mr\Omega^2 e^{j(\Omega t + \delta)} + \kappa K\rho_T(t)e^{j(\alpha-\alpha_S)}$$

$$\alpha = \arctan(y/x) \qquad z = x + jy \qquad j = \sqrt{-1} \qquad \dot{} = \frac{d}{dt}$$

where z is the rotor lateral displacement combined in one complex number, x is horizontal, y is vertical displacement of the rotor in stationary coordinates

K, M are rotor modal stiffness and mass, respectively

D_s is external damping

D is the rotor surrounding fluid radial damping

λ is the fluid circumferential average velocity ratio (lambda)

Ω is rotor rotative speed (omega)

m, r, δ are the rotor modal unbalance, mass radius and angular orientation (delta), respectively

$\rho_T(t)$ is rub-related, thermally-induced shaft bow expressed in coordinates rotating with the shaft. This bow is a function of time t (rho sub $_T$)

κ is a modal factor which depends on the axial location of the rubbing area (kappa)

α_s is a constant angle related to the shaft twist between the rubbing section and the modal mass location (alpha sub s)

The thermal bow occurs at the angular location of the high spot (with a small shift due to friction angle). The new thermal, bow-related unbalance adds vectorially to the original

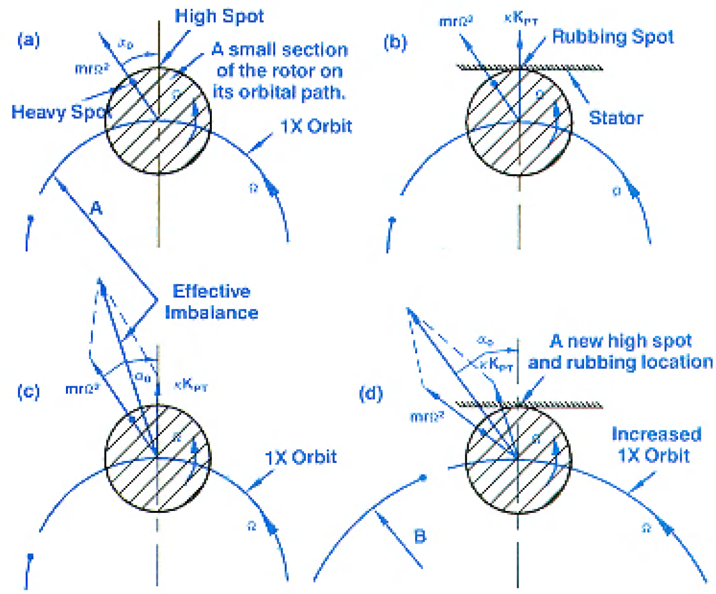

Figure 1

At a constant rotative speed, Ω, below the first balance resonance, the phase lag, α_0, between the heavy spot (original unbalance force) and high spot is less than 90° (a). Due to the rub, the shaft bows in the direction of the high spot and a new unbalance force, F occurs (b). The original and rub-related unbalances add together, producing the "effective" unbalance force (c). At a constant rotative speed, Ω, the phase lag between the effective force and response (high spot) must be constant. This means that the shaft has to rotate, yielding a new high spot (d). Note, also, the increase of Orbit magnitude at the end of this process.

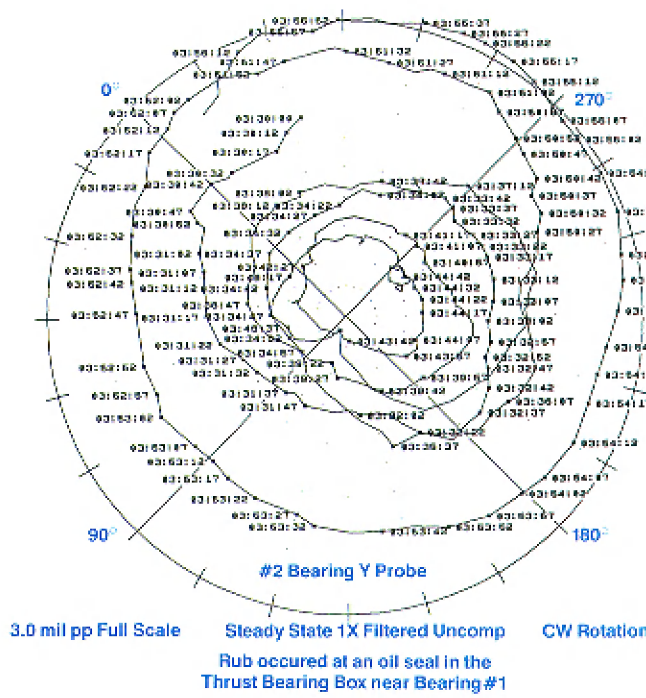

Figure 2

Thermal bow-related rub diagnosis of a turbine rotor: Polar plot of time-trended synchronous (1X) response vector. Rotative speed Ω = 3600 rpm. During 30 minutes, the 1X vector accomplished six full circles.

unbalance. When the rotor runs at a speed below the first balance resonance, the angle between the heavy spot (original unbalance) and high spot (direction of the new rub-related unbalance) is less than 90 degrees. The vectorial sum of them is, therefore, higher than the original unbalance, and the resulting phase angle has to adjust to the phase appropriate for the specific constant speed of rotation. Figure 1 illustrates this effect. The model Equation (1) explains the phenomenon mathematically.

Assume that the thermal bow ρ_T is a known, slowly varying function of time. Assume, also, that there was no rubbing during startup of the rotor up to the rotative speed Ω. At the instant just before the rub occurred, the rotor vibrations described by Equation (1) were synchronous due to original unbalance only ($\rho_T\ (0)=0$):

$$z(t)\big|_0 = A_0 e^{j(\Omega t+\delta+\alpha_0)} \tag{2}$$

where the amplitude and the phase are as follows: (3)

$$A_0 = \frac{mr\Omega^2}{\sqrt{(K-M\Omega^2)^2+\Omega^2[D_s+D(1-\lambda)]^2}},\quad \alpha_0 = \arctan\frac{\Omega[D_s+D(1-\lambda)]}{M\Omega^2-K}$$

The angle $\delta+\alpha_0$ describes the high spot orientation, as measured by the Keyphasor® transducer.

At the moment when the rub occurs, the synchronous vibrations start changing because the force balance, Equation (1), acquires a new, synchronously rotating force related to the thermally-induced shaft bow unbalance:

$$F = \kappa K \rho_T(t) e^{j(\Omega t+\delta-\alpha_s+\alpha_0)},\qquad \text{because } \alpha = \Omega t+\delta+\alpha_0$$

Therefore, at this moment, the rotor response becomes:

$$z\big|_1 = A_0 e^{j(\Omega t+\delta+\alpha_0)} + (A_1\big|_1) e^{j(\Omega t+\delta-\alpha_s+2\alpha_0)} = B_1 e^{j(\Omega t+\delta+\alpha_0+\beta_1)} \tag{4}$$

where

$$A_1\big|_1 = \frac{\kappa K(\rho_T\big|_1)}{\sqrt{(K-M\Omega^2)^2+\Omega^2[D_s+D(1-\lambda)]^2}}$$

is the amplitude of the response to the shaft bow-related unbalance force. The corresponding response phase is the same, α_0, as defined previously. Two elements of the response (4) can be vectorially added. The newly-acquired amplitude, B_1 and phase β_1, of the rotor total synchronous response are as follows:

$$B_1 = \sqrt{A_0^2+(A_1\big|_1)^2+2(A_1\big|_1)A_0\cos(\alpha_0-\alpha_s)} \tag{5}$$

$$\beta_1 = \arctan\frac{\tan(\alpha_0-\alpha_s)}{1+\dfrac{A_0}{(A_1\big|_1)\cos(\alpha_0-\alpha_s)}}$$

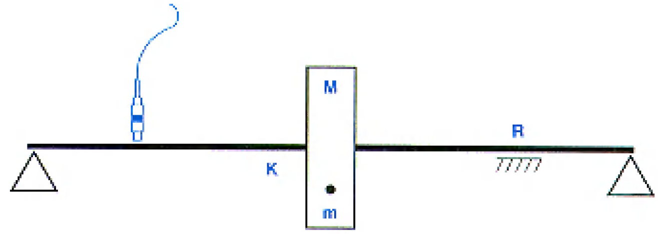

Figure 3.
Schematic of a rotor rubbing at the axial location, R.

It can be seen that, in comparison to that of Equation (2), the synchronous amplitude B_1 in Equation (4) is magnified by some factor, and the phase lag becomes higher by the amount β_1.

At the next moment of the continuously rubbing rotor, (these moments, denoted by subscripts $|_i$, $i=0,1,2,\ldots$, belong to a different, much slower, time scale than t) the synchronous vibrations are: (6)

$$z(t)\big|_2 = A_0 e^{j(\Omega t+\delta+\alpha_0)} + (A_1\big|_2) e^{j(\Omega t+\delta-\alpha_s+2\alpha_0+\beta_1)} = B_2 e^{j(\Omega t+\delta+\alpha_0+\beta_2)}$$

where

$$(A_1\big|_2) = (A_1\big|_1)\frac{(\rho_T(t)\big|_2)}{(\rho_T(t)\big|_1)} \tag{7}$$

and the newly-acquired 1X amplitude and phase are:

$$B_2 = \sqrt{A_0^2+(A_1\big|_2)^2+2A_0(A_1\big|_2)\cos(\alpha_0-\alpha_s+\beta_1)} \tag{8}$$

$$\beta_2 = \arctan\frac{\tan(\alpha_0-\alpha_s+\beta_1)}{1+\dfrac{A_0}{(A_1\big|_2)\cos(\alpha_0-\alpha_s+\beta_1)}} \tag{9}$$

At this moment, further amplitude and phase lag growths are noted.

By continuing the calculating process, at any next moment "i," the rotor synchronous vibrations can be calculated based on the previous moment "i–1," and will have the following form: (10)

$$z(t)\big|_i = A_0 e^{j(\Omega t+\delta+\alpha_0)} + (A_1\big|_i) e^{j(\Omega t+\delta-\alpha_s+2\alpha_0+\beta_{i-1})} = B_i e^{j(\Omega t+\delta+\alpha_0+\beta_i)},\ i=1,2,3,\ldots$$

where

$$A_1\big|_i = \frac{\kappa[\rho_T(t)\big|_i]}{\sqrt{(K-M\Omega^2)^2+\Omega^2[D_s+D(1-\lambda)]^2}} \tag{11}$$

$$B_i = \frac{}{\sqrt{A_0^2+(A_1\big|_i)^2+2A_0(A_1\big|_i)\cos(\alpha_0-\alpha_s+\beta_{i-1})}} \tag{12}$$

$$\beta_i = \arctan \frac{\tan(\alpha_0 - \alpha_S + \beta_{i-1})}{1 + \dfrac{A_0}{(A_1|_i)\cos(\alpha_0 - \alpha_S + \beta_{i-1})}} \qquad (13)$$

Equation (10) describes the rotor spiraling/oscillating mode of synchronous vibrations. The full rotation of the phase is reached for the value of "i" when $\beta_i \geq 360°$. The number of steps "i" in the slow time scale needed to accomplish the full rotation depends on the initial phase lag, α_0, which, in turn, depends on the closeness of the rotative speed to the first balance resonance frequency, $\sqrt{K/M}$. The closer the value of Ω is to $\sqrt{K/M}$, the higher the original phase lag α_0 is, and the faster the full rotation is accomplished. Numerical examples below illustrate this effect.

Numerical examples

Example 1

In this numerical example, it is assumed that the original unbalance-to-thermally-induced unbalance amplitude ratio is constant:

$$\frac{\kappa K \rho_T|_i}{mr\Omega^2} = A_i/A_0 = 2$$

Assume that $\alpha_0 = -40°$, $\delta = 0$, $\alpha_s = 1°$.

The rotative speed, Ω, to the system natural frequency ratio can be expressed in terms of the phase lag α_0, and the damping factor $\zeta = [D_S + D(1-\lambda)]/2\sqrt{K/M}$:

$$\frac{\Omega}{\sqrt{K/M}} = \frac{\zeta}{\tan\alpha_0} + \sqrt{1 + \left(\frac{\zeta}{\tan\alpha_0}\right)^2} = -1.19\zeta + \sqrt{1 + (1.19\zeta)^2}$$

For the damping factor $\zeta = 0.02$, the frequency ratio is $\Omega/\sqrt{K/M} = 0.98$.

Calculation results are presented in Table 1 and Figure 4.

As can be seen, the full 360° rotation of the response phase was accomplished in twelve steps (i). After an initial increase between steps 0 and 1, the response amplitude is slowly variable.

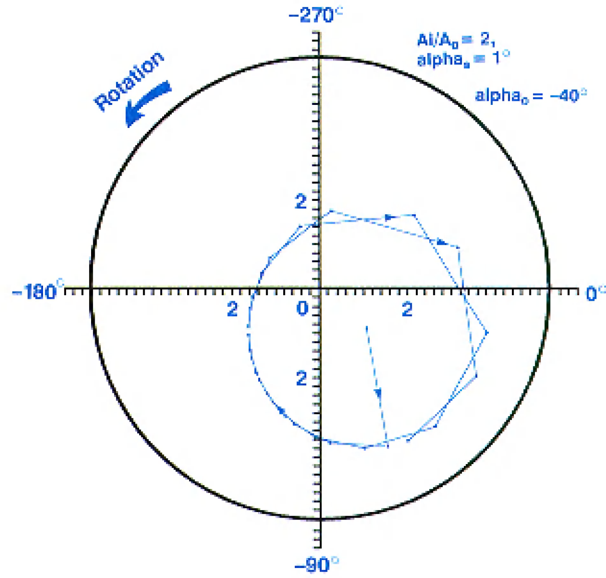

Figure 4
Polar plot of the rotor synchronous (1X) response. One full rotation of the phase accomplished in twelve steps "i" (Example 1).

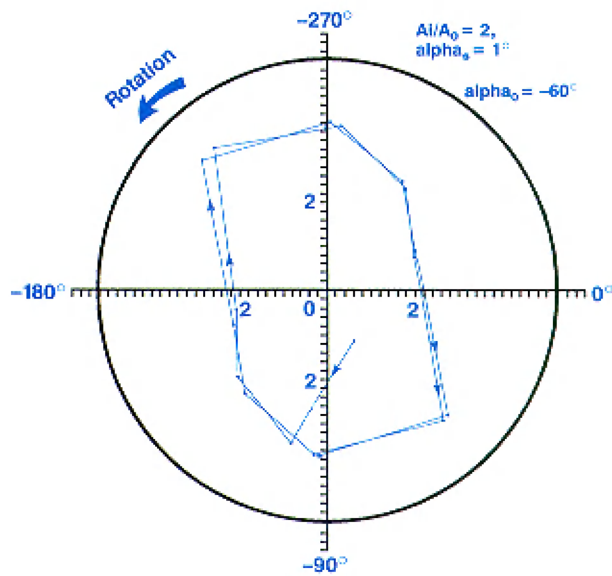

Figure 5
Polar plot of the rotor synchronous (1X) response. One full rotation of the phase accomplished in six steps "i" (Example 2).

Step i	0	1	2	3	4	5	6	7	8	9
B_i/A_o	1	2.83	2.54	2.27	2.03	1.82	1.62	1.43	1.23	1.04
$\alpha_0 + \beta_i$	–40°	–67.6°	–87.1°	–101.9°	–142.2°	–125.3°	–136.5°	–149.4°	–166.7°	–196.0°

Step i	10	11	12	13	14	15	16	17	18
B_i/A_o	1.08	1.97	2.87	2.97	2.73	2.44	2.18	1.95	1.75
$\alpha_0 + \beta_i$	–252.6°	–322.8°	–15.7°	–51.1°	–75.3°	–92.8°	–106.5°	–118.2°	–129.2°

Table 1
Rubbing rotor to non-rubbing rotor 1X response amplitude ratio and its phase as functions of "slow" time steps "i"

Example 2

In comparison to Example 1, in this example the phase angle lag, α_0, is higher. That is, it is assumed that the rotative speed is closer to the resonance.

$$\alpha_0 = -60°$$

$$\frac{\Omega}{\sqrt{K/M}} = -0.58\zeta + \sqrt{1 + (0.58\zeta)^2}$$

For the damping ratio $\zeta = 0.02$, $\Omega/\sqrt{K/M} = 0.99$.

The results are presented in Table 2 and Figure 5.

In this example, the full 360° rotation of the response phase was accomplished in less than six steps (i). The Polar plots for both Examples 1 and 2 are very similar; they differ, however, by the time period of the full rotation. The closer the rotative speed is to resonance, the faster the full phase rotation is accomplished.

Step i	0	1	2	3	4	5	6	7	8
B_i/A_o	1	2.63	2.03	1.48	1.05	1.33	2.68	2.95	2.44
$\alpha_0+\beta_i$	–60°	–101.6°	–133.9°	–166.2°	–215.0°	–302.3°	–21.4°	–75.0°	–112.6°

Step i	9	10	11	12	13	14	15	16	17
B_i/A_o	1.84	1.31	1.00	1.74	2.92	2.82	2.25	1.66	1.17
$\alpha_0+\beta_i$	–143.8°	–178.8°	–239.6°	–330.6°	–41.0°	–88.3°	–122.9°	–153.9°	–193.8°

Step i	18	19	20	21	22	23	24
B_i/A_o	1.06	2.23	3.00	2.65	2.05	1.49	1.06
$\alpha_0+\beta_i$	–268.7°	–356.3°	–58.2°	–100.3°	–132.8°	–164.9°	–212.6°

Table 2
Rubbing rotor to non-rubbing rotor 1X response amplitude ratio and its phase as functions of "slow" time steps "i"

Example 3

In this example, the rub-related thermal bow unbalance is considered increasing, according to the following relationship:

$$\frac{\kappa K \rho_T|_i}{mr\Omega^2} = A_i/A_0 = 2 + 0.1i$$

The remaining parameters are the same as in Example 2.

The results are presented in Table 3 and Figure 6.

The full 360° phase rotation is accomplished in less than six steps, similar to Example 2. The amplitude is slowly increasing, demonstrating the spiraling effect.

Step i	0	1	2	3	4	5	6	7	8	9
B_i/A_o	1	2.73	2.20	1.69	1.40	2.07	3.41	3.60	3.08	2.46
$\alpha_0+\beta_i$	–60°	–102.3°	–137.0°	–174.8°	–232.7°	–316.6°	–29.0°	–82.0°	–124.3°	–165.9°

Step i	10	11	12	13	14	15	16	17	18	19
B_i/A_o	2.03	2.44	3.77	4.28	3.84	3.16	2.66	2.96	4.24	4.90
$\alpha_0+\beta_i$	–220.6°	–291.3°	–11.8°	–69.8°	–116.6°	–61.3°	–215.8°	–288.5°	–2.3°	–62.6°

Table 3
Rubbing rotor to non-rubbing rotor 1X response amplitude ratio and its phase as functions of "slow" time steps "i"

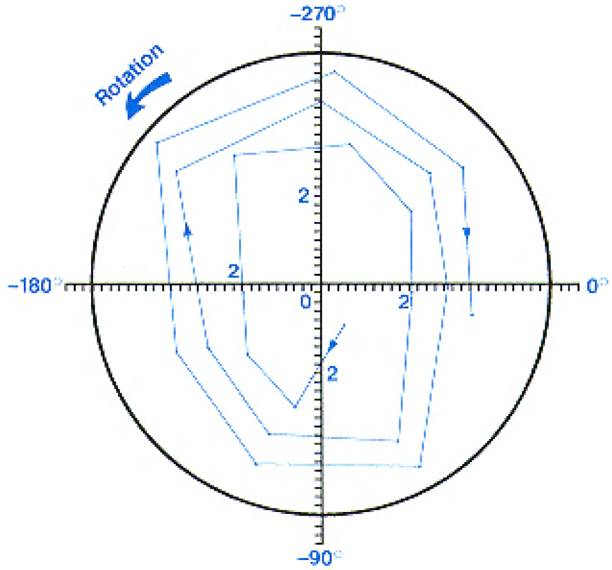

Figure 6
Polar plot of the rotor synchronous (1X) response for the case when the rub-induced bow is an increasing function of time (Example 3).

Conclusions

The model of the thermal rub phenomenon presented in this article is certainly very simplified. It was assumed, in particular, that the rub occurs at some axial location, and its effect on the first mode symmetric rotor behavior was limited to rotor bowing. The amount of the rotor thermally-induced bow was not quantified, nor correlated with the rubbing rotor-to-stator contact frictional and local expansion effects. It was only qualitatively shown that, at a constant rotative speed, the thermally-induced rotor bow, as a result of rubbing, creates additional unbalance, and, consequently, causes phase lagging as the rubbing spot continuously changes.

The assumption about the rotor symmetry (isotropy) and limitation to only one lateral mode provides another simplification. Both of these latter assumptions can easily be alleviated without losing any qualitative features. In particular, if the rotor speed is in the range of the second mode, slightly lower than the second balance resonance, and if the rub occurs at the rotor axial location in the area of the highest unbalance, then the angular situation between the original unbalance and rub-related unbalance is practically the same as in the simple one-mode example considered above. Therefore, the rubbing rotor behavior will be very similar, as discussed in this article.■

References:

1. Taylor, H. D., "Rubbing Shafts Above and Below the Resonance Speed (Critical Speed)," General Electric Company, R-16709, Schenectady, New York, April 1924.
2. Newkirk, B. L., "Shaft Rubbing. Relative Freedom of Rotor Shafts From Sensitiveness to Rubbing Contact When Running Above Their Critical Speeds," Mechanical Engineering, v. 48, No. 8, August 1926, pp. 830-832.

Notation

$A_1\|_i, A_0$	Rotor synchronous response amplitude with and without thermal rub effects
B_i	Rotor synchronous response amplitude affected by rub thermal effect
D, D_s	Fluid damping and shaft modal external damping, respectively
$i = 1,2,...$	Integer describing time instants
$j = \sqrt{-1}$	
K	Rotor modal stiffness
m, r, δ	Unbalance, mass radius and angular orientation, respectively
M	Rotor modal mass
t	Time
x, y	Rotor horizontal and vertical coordinates at modal mass location
$z = x + jy$	Rotor lateral coordinates at the modal mass
$z\|_i$	Rotor synchronous response at instant "i"
α_0	Rotor synchronous response phase without thermal effect
α	Rotor instantaneous angular orientation at modal mass
α_s	Angular twist between the rubbing place and modal mass location
κ	Modal factor
λ	Fluid circumferential average velocity ratio
ρ_T	Rub-related thermally induced shaft bow expressed in rotating coordinates
Ω	Rotative speed